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Free energy fluctuations for bimodal directed polymers in 1+1 dimensions

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By investigating the covariance between the free energy and the entropy, we studied the free energy fluctuations for bimodal directed polymers at finite temperatures. It is proposed that they have a phase transition from the KPZ behavior for $p<p_c$ to the EW behavior for $p>p_c$ for the free energy fluctuations at any finite temperature.

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Directed polymers in a random media has received much attention [1,2]. This study may have a lot of practical applications, e.g., in surface growth phenomena [3], paper rupture [4], spin glasses [5], and high-T_c superconductors [6]. The problem for directed polymers with continuous disorder distribution is equivalent to the Kardar-Parisi-Zhang (KPZ) equation [3,7–9], which described ballistic deposition [10] in surface growth phenomena. When the disorder on the directed path is bimodal distribution, where the bond of the lattice is randomly assigned an energy $\epsilon=0$ with the probability p or an energy $\epsilon=1$ with the probability $1-p$, this problem is called bimodal directed polymers [11–15]. When the probability $p=p_c\approx 0.6447$, the bimodal directed polymers becomes directed percolation (DP) where an infinite fractal cluster of the bonds taking the energy $\epsilon=0$ appears [16]. The directed percolation is relevant to many other fields from interface morphology with quenched noise [17] to fluid invasion of porous media [18]. The problem of directed percolation cannot be described by the KPZ equation [12–15,17,18]. Below the threshold $p<p_c$, it is believed that bimodal directed polymers at finite temperatures belongs to the KPZ universality [13,15]. But, above the threshold $p>p_c$, the behavior of the bimodal directed polymers is much more elusive [11,13,15]. To the best of our knowledge, most studies [11–15] are done for zero temperature, not for finite temperatures. At finite temperatures, the entropy is an important quantity for characterizing the behavior of the directed polymers in a random medium [19,20]. In this paper, by investigating a quantity related to the entropy, we address the problem for the behavior of the free energy fluctuations of bimodal directed polymers at finite temperatures both for $p<p_c$ and $p>p_c$.

In our recent work [19,20], the problem for directed polymers with uniform distribution disorder at finite temperatures has been studied numerically. The behaviors for the internal energy fluctuations and the ensemble energy fluctuations can be understood after investigating the entropy fluctuations. In this paper, it is shown that the derivative of the free energy fluctuations with temperature is related to the covariance between the free energy and the entropy. By investigating this derivative, it is proposed that there is a phase transition at the threshold $p=p_c$ for the free energy fluctuations for bimodal directed polymers at any finite temperature from the KPZ behavior for $p<p_c$ to the Edwards-Wilkinson (EW) behavior for $p>p_c$.

Consider a directed polymer on a hyperpyramid lattice structure with the random energy assigned on each bond. The partition function $G(x,t)$ for directed polymers starting from (0,0) and ending at (x,t) is defined by $G(x,t)=\Sigma_C e^{-E_C/T}$ where E_C is the sum of the bimodal distribution energies on the path C and T is the temperature. The iteration relation for the partition function $G(x,t)$ is

$$G(x,t+1)=G(x-1,t)e^{-\epsilon_l/T}+G(x+1,t)e^{-\epsilon_r/T}, \quad (1)$$

in which ϵ_l and ϵ_r are the energies assigned to the left and right bonds of the point (x,t). The free energy $F(t)$ is given by $F(t)=-T\ln G(t)$, where $G(t)=\Sigma_x G(x,t)$ is the total partition function. We can also define the internal energy $\langle E\rangle$ by

$$\langle E\rangle=\frac{\sum_x\sum_C E_C e^{-E_C/T}}{\sum_x\sum_C e^{-E_C/T}}=T^2\frac{\partial\ln G}{\partial T}. \quad (2)$$

The transverse fluctuation Δx for the directed polymers is $(\overline{\langle x^2\rangle})^{1/2}$ where $\langle x^2\rangle=\Sigma_x x^2 G(x,t)/G(t)$ ($\overline{A}$ is the ensemble average of the quantity A). The free energy fluctuation is $\Delta F=(\overline{F^2}-\overline{F}^2)^{1/2}$.

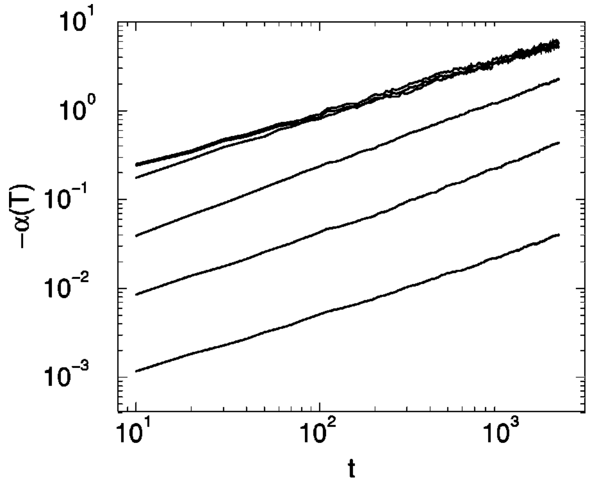

FIG. 1. Plot of the derivative of the free energy fluctuations with temperture $\alpha(T,t)=\partial[(\Delta F)^2]/\partial T$ as a function of time t at different temperatures $T=5,2,1,\frac{1}{4},\frac{1}{20},\frac{1}{30}$ (from bottom to top) for $p=0.3<p_c$.

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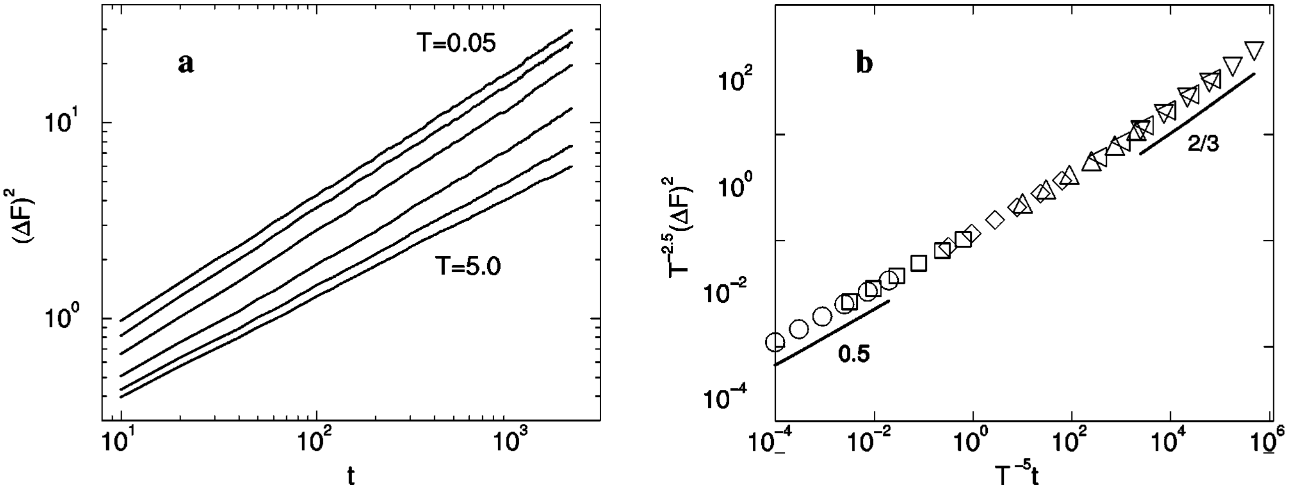

FIG. 2. (a) Plot of the free energy fluctuations $(\Delta F)^2$ as a function of time t at different temperatures $T=5,2,1,\frac{1}{2},\frac{1}{4},\frac{1}{20}$ (from bottom to top) for $p=0.3<p_c$. (b) Scaling plot for the free energy fluctuations $(\Delta F)^2$ shown in (a) for $p=0.3<p_c$ where circle, square, diamond, triangle up, triangle left, and triangle down correspond to different temperatures $T=10,5,2,1,\frac{1}{2},\frac{1}{3}$, respectively.

At zero temperature, the transverse fluctuation takes the KPZ behavior, $\Delta x \sim t^{2/3}$ when $p<p_c$ and take the DP behavior, $\Delta x \sim t^{0.633}$ when $p=p_c$ [11–13,15]. For $p>p_c$, it is also suggested that the transverse fluctuation take the KPZ behavior. (This result is inconclusive [15].) At zero temperature, the free energy fluctuations will degenerate into the fluctuation of the sum of the energy on the optimal path. As a result, the free energy fluctuations have $\Delta F \sim t^{1/3}$ as the KPZ behavior for $p<p_c$ [11,12] and scale as $\Delta F \sim (\ln t)^{1/2}$ [21,22] for $p=p_c$ and reach a constant for $p>p_c$ [13,22]. At infinite temperature, bimodal directed polymers for any p will become random walks. The free energy fluctuations can be predicted by the Edwards-Wilkinson equation [23]. It follows $\Delta F \sim t^{1/4}$.

In order to understand the behaviors of the free energy fluctuations at finite temperatures, we introduce the derivative of the free energy fluctuations with temperature $\alpha(T,t) = \partial(\Delta F)^2/\partial T$. We have

$$\alpha(T,t) = -2[\overline{FS} - \overline{F}\;\overline{S}], \tag{3}$$

where $S=(\langle E\rangle - F)/T$ is the entropy [19,20]. In order to obtain the iteration relation for the internal energy $\langle E\rangle$, we can define $\hat{E}(x,t) \equiv \Sigma_{C(x,t)} E_{C(x,t)} e^{-E_{C(x,t)}/T}/G(t)$. It is clear that $\langle E\rangle = \Sigma_x \hat{E}(x,t)$. The iteration relation for $\hat{E}(x,t)$ is [19,20]:

$$\hat{E}(x,t+1) = [e^{-\epsilon_l/T}\hat{E}(x-1,t)G(t) + e^{-\epsilon_r/T}\hat{E}(x+1,t)$$
$$G(t) + \epsilon_l e^{-\epsilon_l/T}G(x-1,t) + \epsilon_r e^{-\epsilon_r/T}G(x+1,t)]/G(t+1). \tag{4}$$

We present systematic numerical simulations for the free energy fluctuation ΔF and the function $\alpha(T,t)$ based upon Eqs. (1) and (4) with the initial conditions $G(x,0)=\delta_{x,0}$ and $\hat{E}(x,0)=0$. The bond of the hyperpyramid lattice in $1+1$

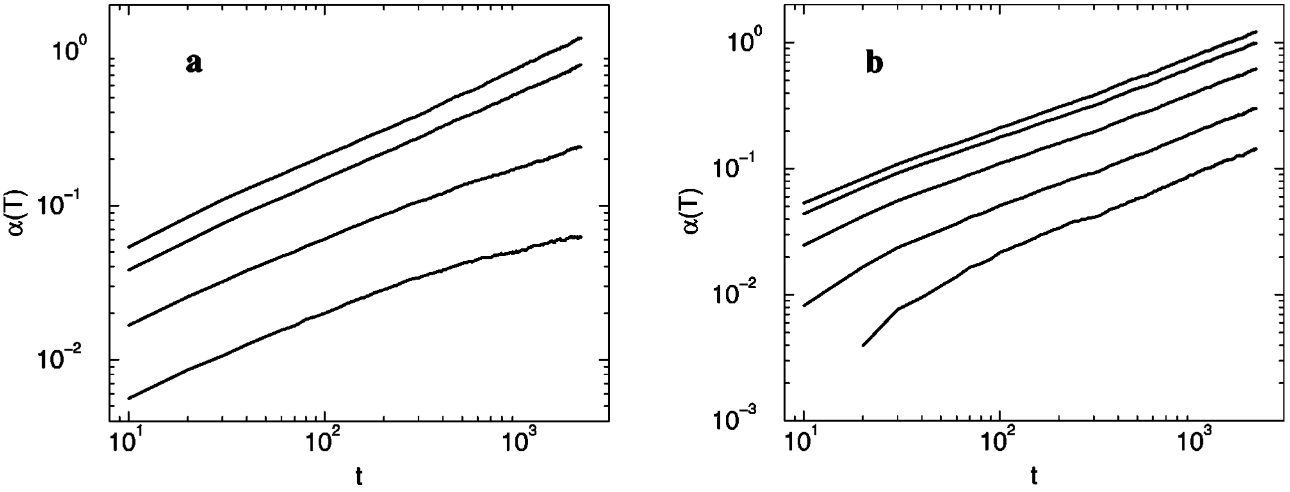

FIG. 3. Plot of the derivative of the free energy fluctuations with temperature $\alpha(T,t) = \partial[(\Delta F)^2]/\partial T$ as a function of time t for $p=0.8>p_c$ at different temperatures: (a) $T=2,1,\frac{1}{2},\frac{1}{4}$ (from bottom to top) and (b) $T=\frac{1}{4},\frac{1}{6},\frac{1}{10},\frac{1}{20},\frac{1}{40}$ (from top to bottom).

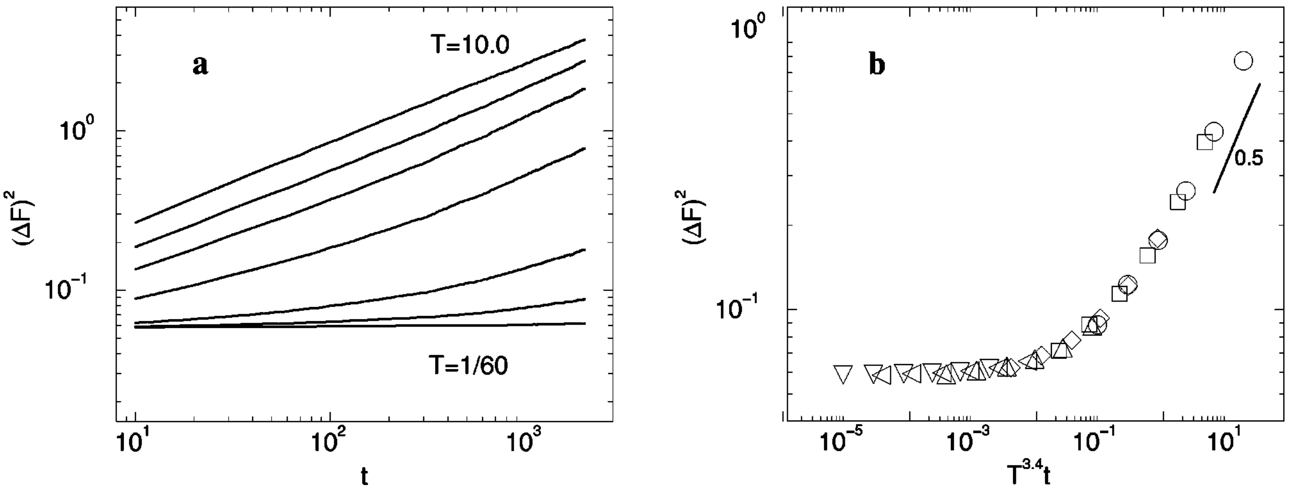

FIG. 4. (a) Plot of the free energy fluctuations $(\Delta F)^2$ as a function of time t at different temperatures $T=10,\frac{1}{2},\frac{1}{4},\frac{1}{10},\frac{1}{20},\frac{1}{60}$ (from top to bottom) for $p=0.8>p_c$. (b) Scaling plot for the free energy fluctuations $(\Delta F)^2$ shown in (a) for $p=0.8<p_c$ where circle, square, diamond, triangle up, triangle left, and triangle down correspond to different temperatures $T=\frac{1}{4},\frac{1}{6},\frac{1}{10},\frac{1}{20},\frac{1}{40},\frac{1}{60}$, respectively.

dimension is randomly assigned an energy $\epsilon=0$ with the probability p or an energy $\epsilon=1$ with the probability $1-p$. We use length up to $t=2000$. Six thousand configurations were collected to take the ensemble average.

For $p=0.3<p_c$, our simulations show that the function $\alpha(T,t)<0$ for any time t (see Fig. 1). Therefore, we have $(\Delta F)_\infty<(\Delta F)_T<(\Delta F)_0$ when the polymer length t is long enough, where $(\Delta F)_T$ is the free energy fluctuations at the temperature T. Our simulations suggest that the free energy fluctuations will crossover from $t^{1/4}$ to $t^{1/3}$ at any finite temperature [see Fig. 2(a)]. Assuming the scaling relation $\Delta F=T^{5/4}f(T^{-5}t)$ for $T>\frac{1}{3}$, the data collapse very well [see Fig. 2(b)].

For $p<p_c=0.6447$, when the value of p is near the threshold p_c, the function $\alpha(T,t)$ may be positive for finite temperatures T at small time t. But, it will decrease with the polymer length t and become negative eventually. For $p>p_c=0.6447$, when the value of p is near the threshold p_c, the function $\alpha(T,t)$ may be positive for finite temperatures at small time t. However, it will become positive when the polymer length t is long.

For $p=0.8>p_c$, our simulations show that the function $\alpha(T,t)>0$ at any time t (see Fig. 3). Therefore, we have $(\Delta F)_0<(\Delta F)_T<(\Delta F)_\infty$ after enough time t. We note that the functions $\alpha(T,t)$ are not monotonic with the temperatures. The function $\alpha(T,t)$ tends to zero at the two limits: $T\rightarrow 0$ or $T\rightarrow\infty$ and reach a maximum at about $T=0.25$. Because the free energy fluctuations (ΔF) scale as $t^{1/4}$ at infinite temperature, (ΔF) must not scale as $t^{1/3}$ at any temperature. It means that the free energy fluctuations (ΔF) do not take the KPZ behaviors when $p>p_c$. For finite temperatures, numerical simulations show that the increase of the free energy fluctuations with the polymer length t becomes faster and faster [see Fig. 4(a)]. We suggest that the free energy fluctuations will crossover from the constant to $t^{1/4}$ for long time t at any finite temperature. We find very good fit with the scaling relations $(\Delta F)=g(T^{3.4}t)$ for $T<0.25$ [see Fig. 4(b)].

At finite temperatures, a phase transition can be observed for the free energy fluctuations from the KPZ behavior $(\Delta F)\sim t^{1/3}$ for $p<p_c$ to the EW behavior, $(\Delta F)\sim t^{1/4}$ for $p>p_c$. This result confirms that the bimodal direccted polymers at finite temperatures for $p<p_c$ belongs to the KPZ universality. But it excludes the possibility [11] that the KPZ universality, holding for $p>p_c$. This suggests that the discreteness of the random medium has a strong effect upon the behavior of the directed polymers.

At the threshold $p=p_c$, an infinite fractal cluster of the bonds taking the energy $\epsilon=0$ appears. The situations will be more complicated. Our simulations show that the free energy fluctuations seem to scale as $t^{1/4}$ for long time t at finite temperatures. But this result is inconclusive and needs further study.

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